

Mathematics

HP COMPUTER CURRICULUM

Linear Equations  
& Systems

TEACHERS ADVISOR

HEWLETT  PACKARD

Hewlett-Packard  
Computer Curriculum Series

**mathematics  
TEACHER'S ADVISOR**

**linear equations  
& systems**

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This material is designed to be used with any Hewlett-Packard system with the BASIC program language such as the 9830A Educational BASIC, and the 2000 and 3000 series systems.

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**MATHEMATICS**

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Hewlett-Packard Computer Curriculum

## INTRODUCTION

This Mathematics Set of the Hewlett-Packard Computer Curriculum Series consists of a set of a Student Book and a corresponding Teacher's Advisor. It was designed to help meet the need for computer-oriented problems in mathematics, providing students an opportunity to use a computer as a problem solving tool to model mathematical concepts.

The materials are designed for flexible use as desired by the individual instructor. The material and exercises in this unit are intended as an "enrichment" experience in the field of linear equations and systems. The unit begins with a discussion of the various forms for a linear equation. The student is taught to derive the equation for a line when given a set of data such as two points on the line, one point and the slope, etc. Other exercises involve applications of this technique. The second section deals with solving linear systems. Two methods are discussed: an Iterative Method for solving low order systems, and the Gaussian Algorithm for higher order systems. The last section is a brief introduction to linear programming. The unit is obviously not a complete study of linear equations and systems and should not compete with your text. The material within each section is sequential, but the sections can be studied independently and in any order. Thus, the material can be used to supplement and enrich your curriculum in any fashion you choose.

The mathematical concepts needed for each exercise are briefly reviewed, but you may want your students to study these in greater detail before attempting the exercises, especially if their background in linear equations is limited. A list of possible references is included at the end of each of the three major sections. These references will also provide additional problems for your students.

The degree of difficulty of the material is dependent upon the amount of assistance which you choose to provide. With no assistance, the better student should be challenged. However, given a good deal of assistance, any second year algebra student should be able to work out the exercises with no great difficulty. The level of the material is determined by the assumption that students taking second year algebra will be quite capable as a group.

The Student Book provides text material and programming exercises for the students. There is a problem analysis, including a suggested approach and a macro flowchart, for each exercise. The Teacher's Advisor contains an example of a program to solve each exercise, micro flowcharts, and a brief discussion of the important elements of the exercise. The micro flowcharts should be given to the students only after they have made an attempt to solve the problem on their own.

For best results, you should study all the solutions until you are certain you have a complete grasp of the general methods. This should be done before assigning the material to the class.

You will undoubtedly think of different programming methods or techniques as you study the example programs. Encourage the students to do the same. There are no *approved* solutions. All solutions are acceptable if they produce the correct results. At this level, there is no need for emphasis on the efficiency of a student's program. The important question is, does it work?

## LINEAR EQUATIONS

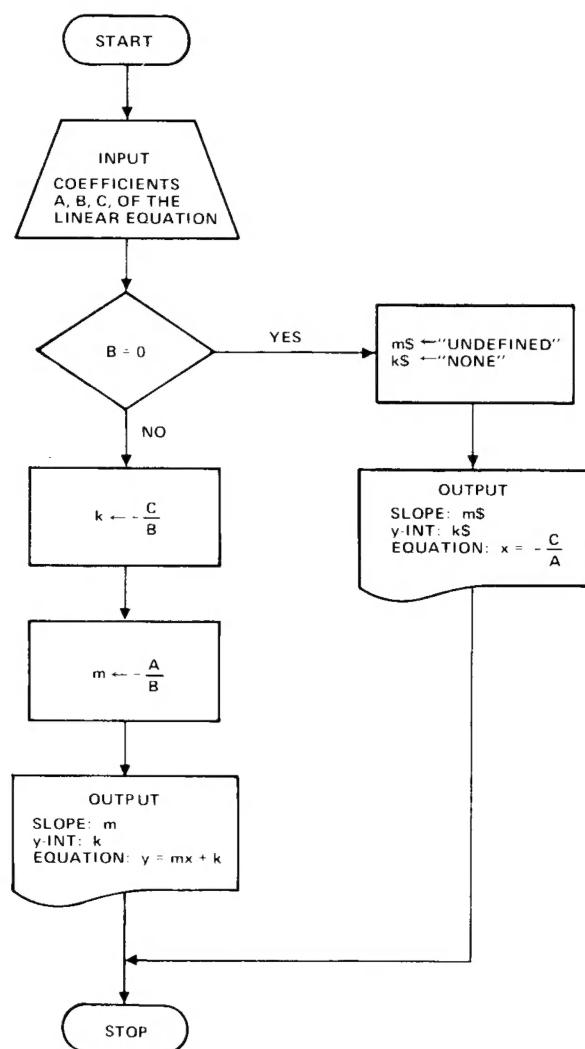
## Exercise 1 — Converting the General Form to Slope-Intercept Form

This program models the definition of slope and y-intercept, and shows the relation between the general form and the y-intercept form of the equation of a line.

The program will not require any sophisticated programming techniques, but it does model those basic concepts involved in the study of linear equations. Make sure the student's program makes provision for consideration of horizontal and vertical lines.

This program can be used to introduce "string variables", if you have them available on your system, by assigning the string "Undefined" to  $m$  if the slope is undefined.

*Micro Flowchart*  
Exercise 1



*Example Program**Exercise 1*

```

10  DIM M$(9)
20  DIM K$(4)
30  REM--THIS PROGRAM TAKES THE COEFFICIENTS OF A LINEAR EQUATION
40  REM--IN GENERAL FORM, AX+BY+C = 0, COMPUTES THE SLOPE AND
50  REM--Y-INTERCEPT OF THE LINE AND PRINTS THE SLOPE-INTERCEPT
60  REM--FORM OF THE EQUATION. ON THE DATA LINE ENTER THE
70  REM--COEFFICIENTS A,B AND C.
80  DATA 3,-2,7,.23,.1,-.6,8,-2,-7,3,0,4,0,2,-5
90  READ A,B,C
100 IF B=0 THEN 170
110 LET K=-C/B
120 LET M=-A/B
130 PRINT "SLOPE:";TAB(15);M
140 PRINT "Y-INT:";TAB(15);K
150 PRINT "EQUATION:";TAB(15);"Y=";M;"X+";K
160 GOTO 220
170 LET M$="UNDEFINED"
180 LET K$="NONE"
190 PRINT "SLOPE:";TAB(15);M$
200 PRINT "Y-INT:";TAB(15);K$
210 PRINT " EQUATION:";TAB(15);"X=";-C/A
220 GOTO 90
230 END

```

RUN

```

SLOPE:      1.5
Y-INT:      3.5
EQUATION:   Y= 1.5      X+ 3.5
SLOPE:      -2.3
Y-INT:      6.
EQUATION:   Y=-2.3      X+ 6.
SLOPE:      4
Y-INT:      -3.5
EQUATION:   Y= 4      X+-3.5
SLOPE:      UNDEFINED
Y-INT:      NONE
EQUATION:   X=-1.33333
SLOPE:      0
Y-INT:      2.5
EQUATION:   Y= 0      X+ 2.5

```

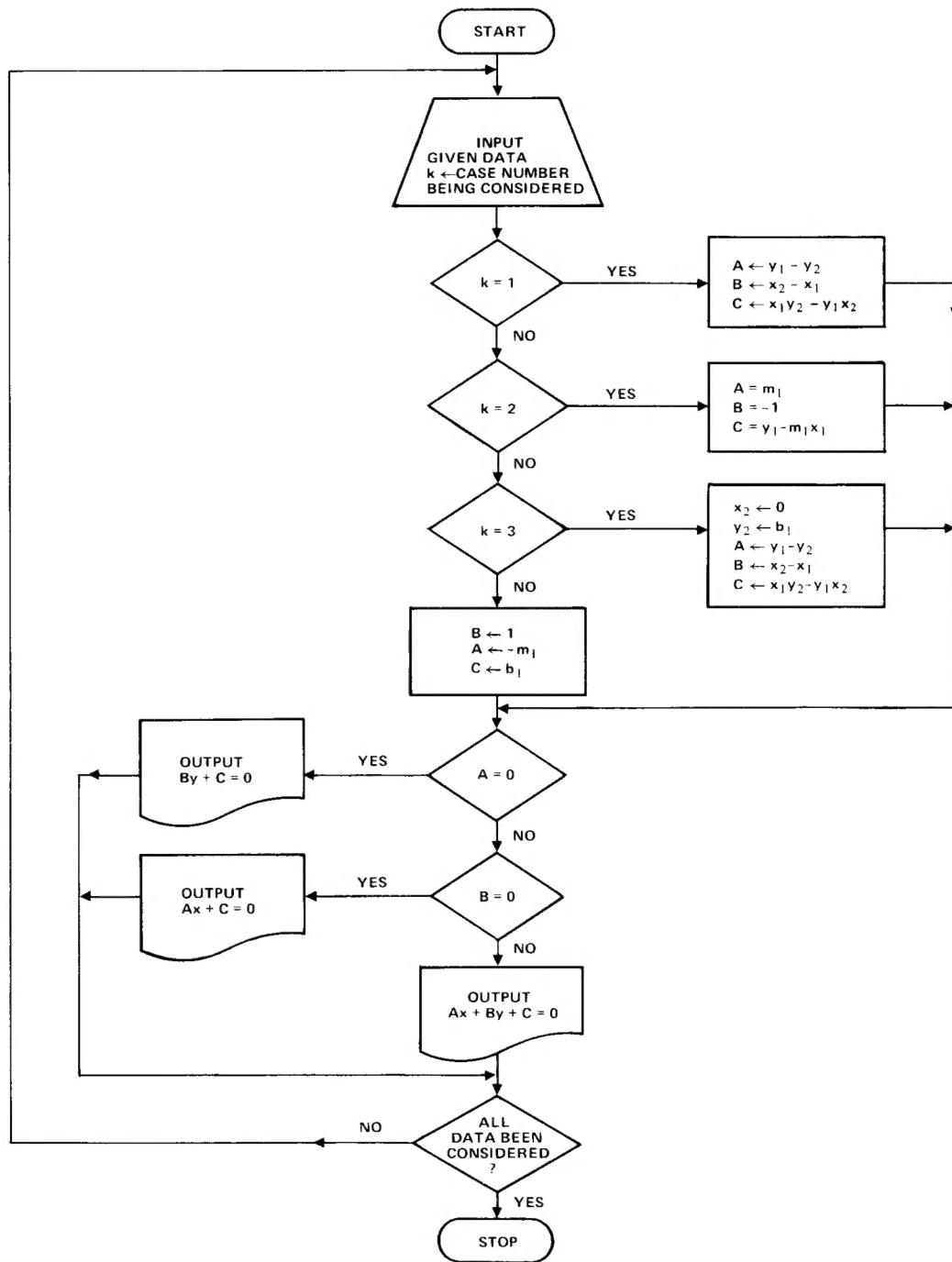
OUT OF DATA IN LINE 90

### Exercise 2 – Finding the General Form from Characteristics of the Line

This exercise makes the student aware of the relationship of the coefficients of the general form and the slope-intercept form of linear equations. Make sure your students understand the development of the different equations, since this is the real value of the exercise.

#### Micro Flowchart

#### Exercise 2.



*Example Program**Exercise 2.*

```
10  REM--A PROGRAM TO PRINT THE GENERAL FORM OF AN EQUATION OF A
20  REM-- LINE GIVEN: CASE (1)TWO POINTS ON THE LINE,
30  REM-- (2)A POINT ON THE LINE AND THE SLOPE OF THE LINE,
40  REM (3)A POINT ON THE LINE AND THE Y-INTERCEPT OF THE LINE.
50  REM--(4) THE SLOPE OF OF THE LINE AND THE Y-INTERCEPT OF THE LINE.
60  REM--ENTER ON THE DATA LINE EACH SET OF DATA TO BE
70  REM-- CONSIDERED PRECEDED BY ITS CASE NUMBER AS NUMBERED ABOVE.
80  DATA 1,2,-1,-3.2,7,1,0,8,-6,0,1,.8,-2,16,-2,1,-1.5,2,-1.5,-8
90  DATA 2,-7.5,-3,-3,3,3.27,-6,-5,4,.875,7.3,4,2.5,0,4,0,2
100 READ K
110 PRINT
120 IF K=1 THEN 230
130 IF K=2 THEN 280
140 IF K=3 THEN 330
150 READ M,B1
160 A=-M
170 B=1
180 C=-B1
190 IF A=0 THEN 370
200 IF B=0 THEN 390
210 PRINT A;"X+";B;"Y+";C;"=0"
220 GOTO 100
230 READ X1,Y1,X2,Y2
240 A=Y1-Y2
250 B=X2-X1
260 C=X1*Y2-Y1*X2
270 GOTO 190
280 READ X1,Y1,M
290 A=M
300 B=-1
310 C=-M*X1+Y1
320 GOTO 190
330 READ X1,Y1,B
340 X2=0
350 Y2=B
360 GOTO 240
370 PRINT B;"Y+";C;"=0"
380 GOTO 100
390 PRINT A;"X+";C;"=0"
400 GOTO 100
410 END
```

RUN

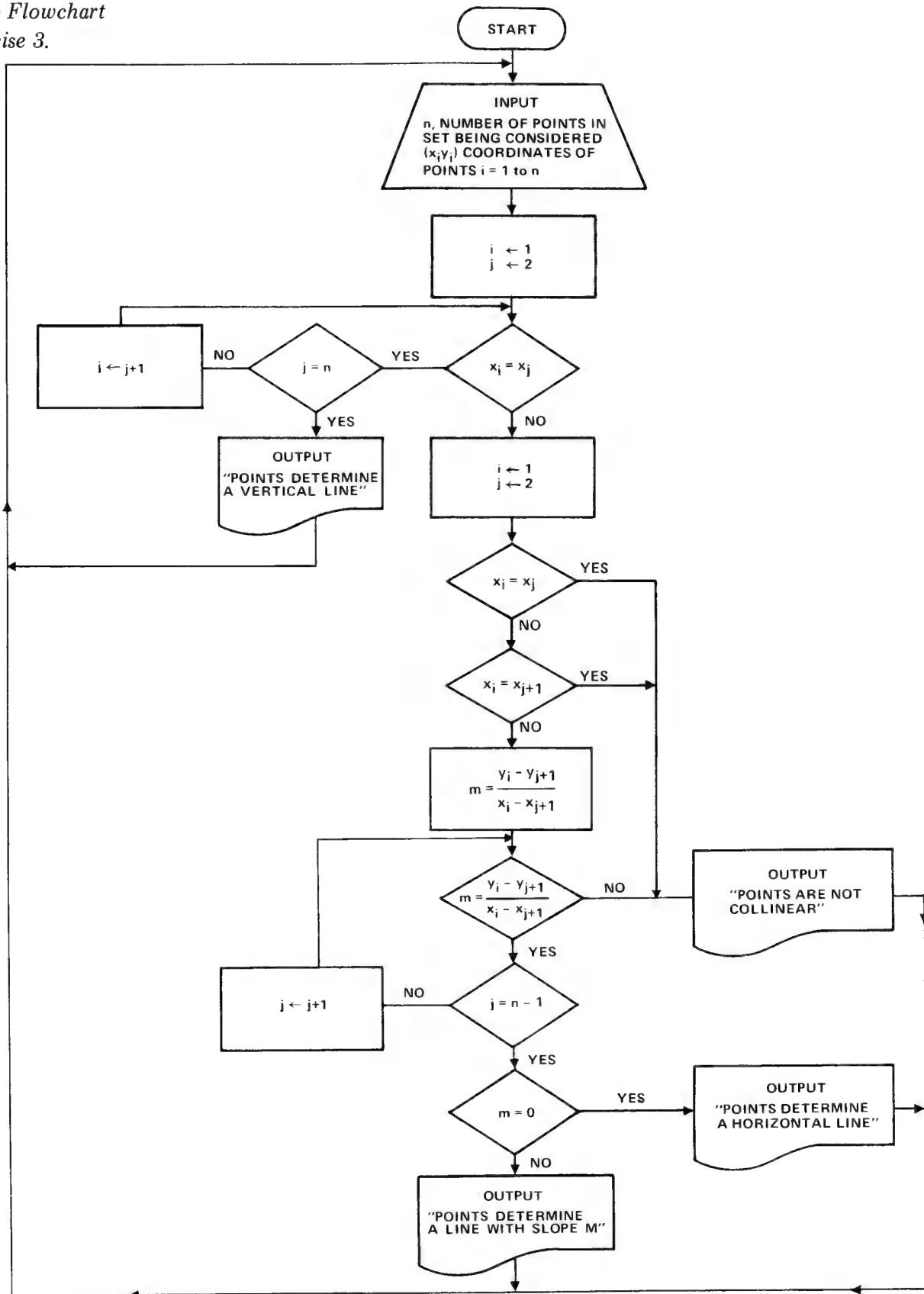
```
-8      X+-5.2          Y+ 10.8      =0
 8      X+-6      Y+ 48      =0
15.2      Y+ 30.4      =0
10      X+ 15      =0
-3      X+-1      Y+-25.5      =0
-1      X+-3.27      Y+-16.35      =0
-.875      X+ 1      Y+-7.3      =0
-2.5      X+ 1      Y+ 0      =0
 1      Y+-2      =0
OUT OF DATA IN LINE 100
```

### Exercise 3 – Determining Collinearity

Exercises 3-6 are really sub-programs of Exercise 7. The students should fully understand these four exercises before attempting Exercise 7.

## Micro Flowchart

## Exercise 3.



*Example Program**Exercise 3.*

```

10 REM--A PROGRAM TO DETERMINE IF A SET OF POINTS IS COLLINEAR.
11 REM-- ENTER ON THE DATA LINE N, THE NUMBER OF POINTS IN THE SET
12 REM--FOLLOWED BY THE COORDINATES OF THE QINTS.
20 DATA 4,-3,-2,0,0,6,4,3,2,3,-1,5,3,5,4.5,5,4,2,-5,-5,-10,2,-5,3,6
21 DATA 4,1,2,-5,-10,2,-5,3,6,4,2,0,2,-3.5,2,8,2,-7
30 READ N
40 FOR I=1 TO N
50 READ X[I],Y[I]
60 NEXT I
80 I=1
90 J=2
100 IF X[I]=X[J] THEN 190
110 I=1
120 J=2
121 IF X[I]=X[J] THEN 150
122 IF X[I]=X[J+1] THEN 150
130 M=(Y[I]-Y[J])/(X[I]-X[J])
140 IF M=(Y[I]-Y[J+1])/(X[I]-X[J+1]) THEN 220
150 PRINT "POINTS ARE NOT COLLINEAR"
180 GOTO 30
190 IF J=N THEN 250
200 J=J+1
210 GOTO 100
220 IF J=N-1 THEN 270
230 J=J+1
240 GOTO 140
250 PRINT "POINTS DETERMINE A VERTICAL LINE"
260 GOTO 30
270 IF M=0 THEN 300
280 PRINT "POINTS DETERMINE A LINE WITH THE SLOPE "3M
290 GOTO 30
300 PRINT "POINTS DETERMINE A HORIZONTAL LINE"
310 GOTO 30
320 END

```

RUN

```

POINTS DETERMINE A LINE WITH THE SLOPE .666667
POINTS DETERMINE A HORIZONTAL LINE
POINTS ARE NOT COLLINEAR
POINTS ARE NOT COLLINEAR
POINTS DETERMINE A VERTICAL LINE

OUT OF DATA IN LINE 30

```

**Exercise 4 – Finding the Length and Midpoint of a Given Line**

This exercise is relatively simple but should still be completed in preparation for Exercise 7.

The flowchart in the Student Book is detailed enough to produce the following program.

*Example Program**Exercise 4.*

```
10 REM--A PROGRAM TO DETERMINE THE LENGTH AND MID-POINT OF THE
11 REM--SEGMENT DETERMINED BY TWO POINTS ON THE CARTESIAN PLANE.
12 REM--ENTER THE COORDINATES OF THE POINTS ON THE DATA LINE.
20 DATA -3,5,2,-7,-7,-1,8,5,-5,7,3.5,7,-4,-6.25,-4,4.8
30 READ X1,Y1,X2,Y2
40 D=SQR((X2-X1)↑2+(Y2-Y1)↑2)
50 M=(X2+X1)/2
60 N=(Y2+Y1)/2
70 PRINT "LENGTH OF SEGMENT P1,P2 IS";D
80 PRINT "MID-POINT, (M,N) ,OF SEGMENT P1,P2 IS (";M;"",";N;"")"
85 GOTO 30
90 END
```

RUN

```
LENGTH OF SEGMENT P1,P2 IS 13
MID-POINT, (M,N) ,OF SEGMENT P1,P2 IS (-.5      , -1      )
LENGTH OF SEGMENT P1,P2 IS 16.1555
MID-POINT, (M,N) ,OF SEGMENT P1,P2 IS ( .5      , 2      )
LENGTH OF SEGMENT P1,P2 IS 8.5
MID-POINT, (M,N) ,OF SEGMENT P1,P2 IS (-.75     , 7      )
LENGTH OF SEGMENT P1,P2 IS 11.05
MID-POINT, (M,N) ,OF SEGMENT P1,P2 IS (-4      , -4.725  )
```

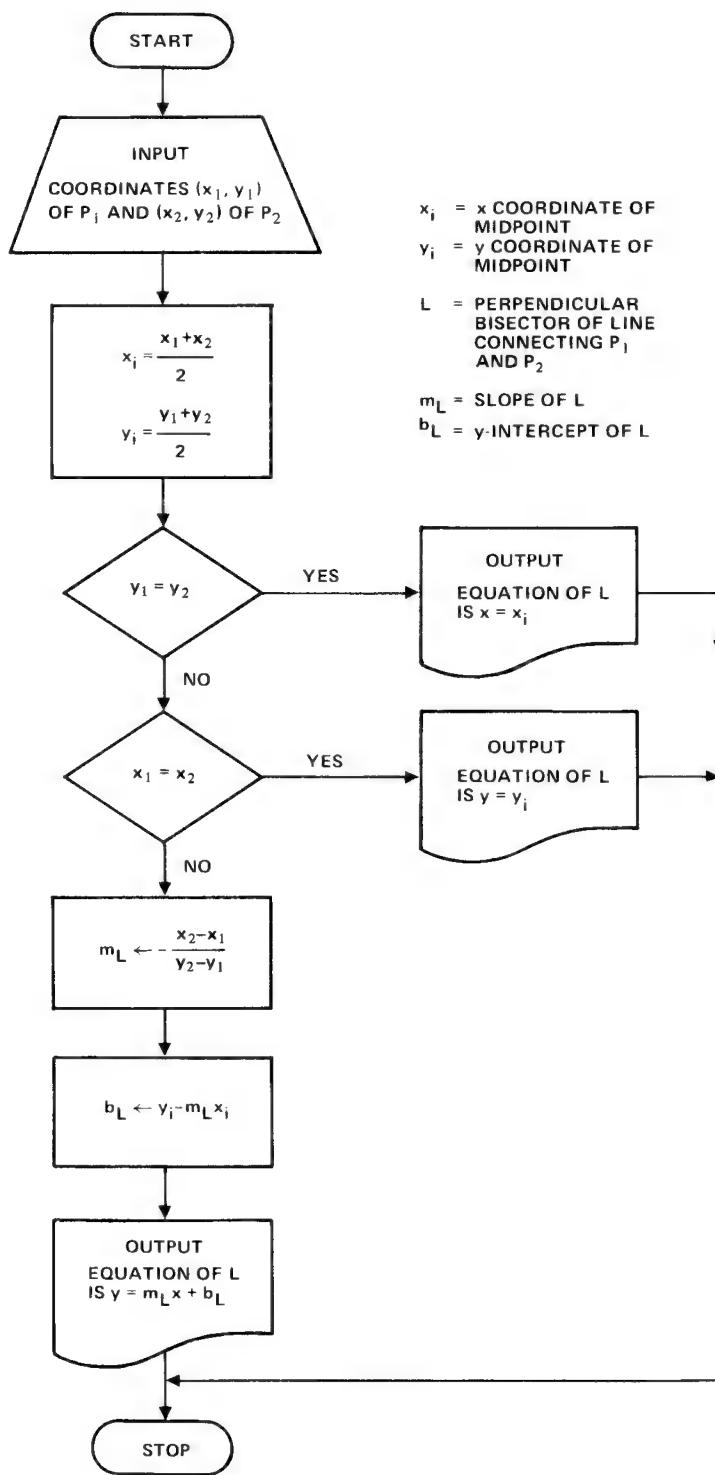
OUT OF DATA IN LINE 30

## Exercise 5 – Equation for the Perpendicular Bisector of a Line

The student will not encounter any new concepts in this exercise. Concepts of previous exercises are applied.

Micro Flowchart

Exercise 5.



*Example Program*

## Exercise 5.

```

10 REM--A PROGRAM TO FIND THE EQUATION OF THE PERPENDICULAR BI-SECTOR
20 REM--OF A LINE SEGMENT DETERMINED BY TWO POINTS IN THE CARTESIAN
30 REM--PLANE. ON THE DATA LINE ENTER THE COORDINATES OF
40 REM--THE TWO POINTS.
50 DATA -7,2,5,-6,7,0,7,17,-11.5,-3,1,-3
60 READ X1,Y1,X2,Y2
70 X=(X1+X2)/2
80 Y=(Y1+Y2)/2
90 IF Y1=Y2 THEN 150
100 IF X1=X2 THEN 170
110 M=-(X2-X1)/(Y2-Y1)
120 B=Y-M*X
130 PRINT "EQUATION OF LINE L IS Y="";M;"*X+"";B
140 GOTO 180
150 PRINT "THE EQUATION OF LINE L IS X="";X
160 GOTO 180
170 PRINT "THE EQUATION OF LINE L IS Y="";Y
180 GOTO 60
190 END

```

RUN

```

EQUATION OF LINE L IS Y= 1.5          *X+-0.5
THE EQUATION OF LINE L IS Y= 8.5
THE EQUATION OF LINE L IS X=-5.25

```

OUT OF DATA IN LINE 60

## Exercise 6 — The Intersection of Two Lines

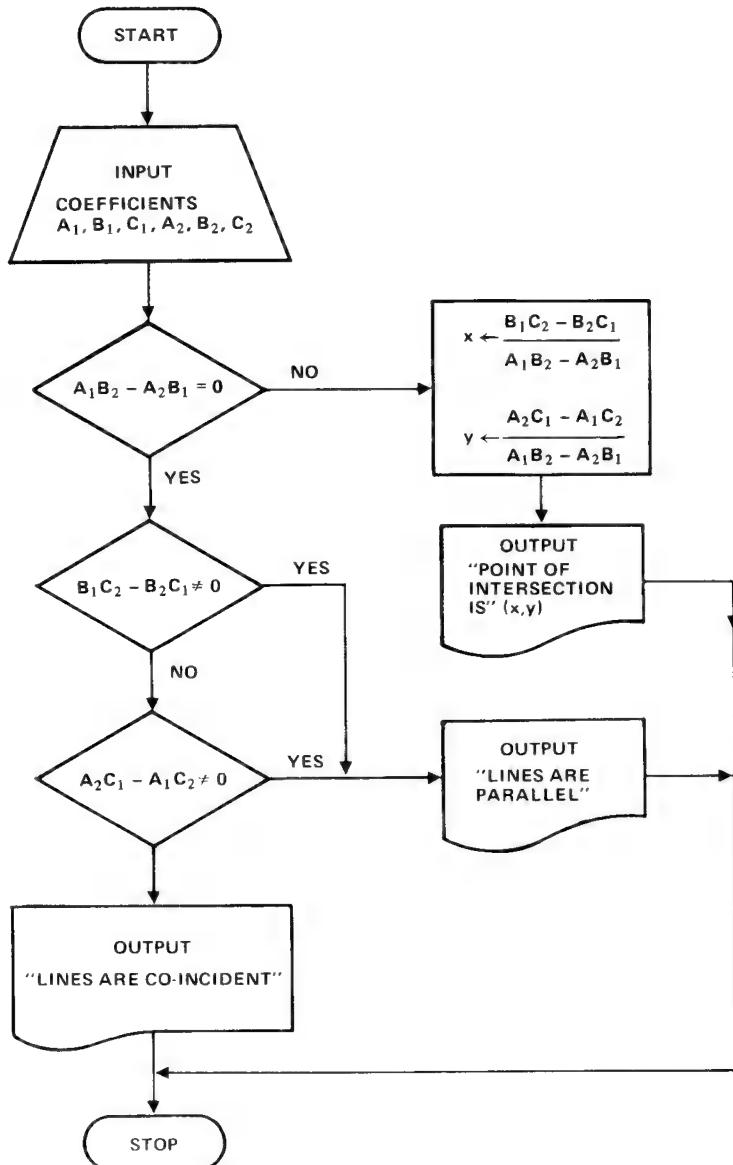
This exercise deals only with the solution of a second order linear system. Once again this is a sub-program of Exercise 7. Higher order systems will be considered in the section on Linear Systems.

It is very important that your students fully understand the development of the expressions which represent the coordinates of the point of intersection. Encourage them to consult reference works. Not much value comes from just having the computer evaluate the expressions

$$\frac{B_1 C_2 - B_2 C_1}{A_1 B_2 - A_2 B_1} \text{ and } \frac{A_2 C_1 - A_1 C_2}{A_1 B_2 - A_2 B_1}$$

The value of the exercise comes from understanding the derivation of the expressions and the conditions under which unique solutions occur.

*Micro Flowchart*  
Exercise 6.



*Example Program*

## Exercise 6.

```
10 REM--A PROGRAM TO DETERMINE THE COORDINATES OF
11 REM--THE POINT OF INTERSECTION OF TWO
20 REM--LINES, GIVEN THEIR EQUATIONS IN GENERAL FORM.
30 REM--ON THE DATA LINES ENTER THE COEFFICIENTS OF THE TWO
40 REM--LINEAR EQUATIONS.
50 DATA 2,-1,3,1,1,-9,.25,1,-2,1,4,2,2,-2,-99,.5,.6,-14,1,2,-3,4,8,-12
60 READ A1,B1,C1,A2,B2,C2
70 IF A1*B2-A2*B1=0 THEN 110
80 PRINT "THE POINT OF INTERSECTION IS (";(B1*C2-B2*C1)/(A1*B2-A2*B1);"
90 PRINT ",";(A2*C1-A1*C2)/(A1*B2-A2*B1);")"
100 GOTO 60
110 IF B1*C2-B2*C1 <> 0 THEN 150
120 IF A2*C1-A1*C2 <> 0 THEN 150
130 PRINT "LINES ARE CO-INCIDENT"
140 GOTO 60
150 PRINT "LINES ARE PARALLEL"
160 GOTO 60
170 END
```

RUN

```
THE POINT OF INTERSECTION IS ( 2      , 7      )
LINES ARE PARALLEL
THE POINT OF INTERSECTION IS ( 39.7273      ,-9.77273      )
LINES ARE CO-INCIDENT

OUT OF DATA IN LINE 60
```

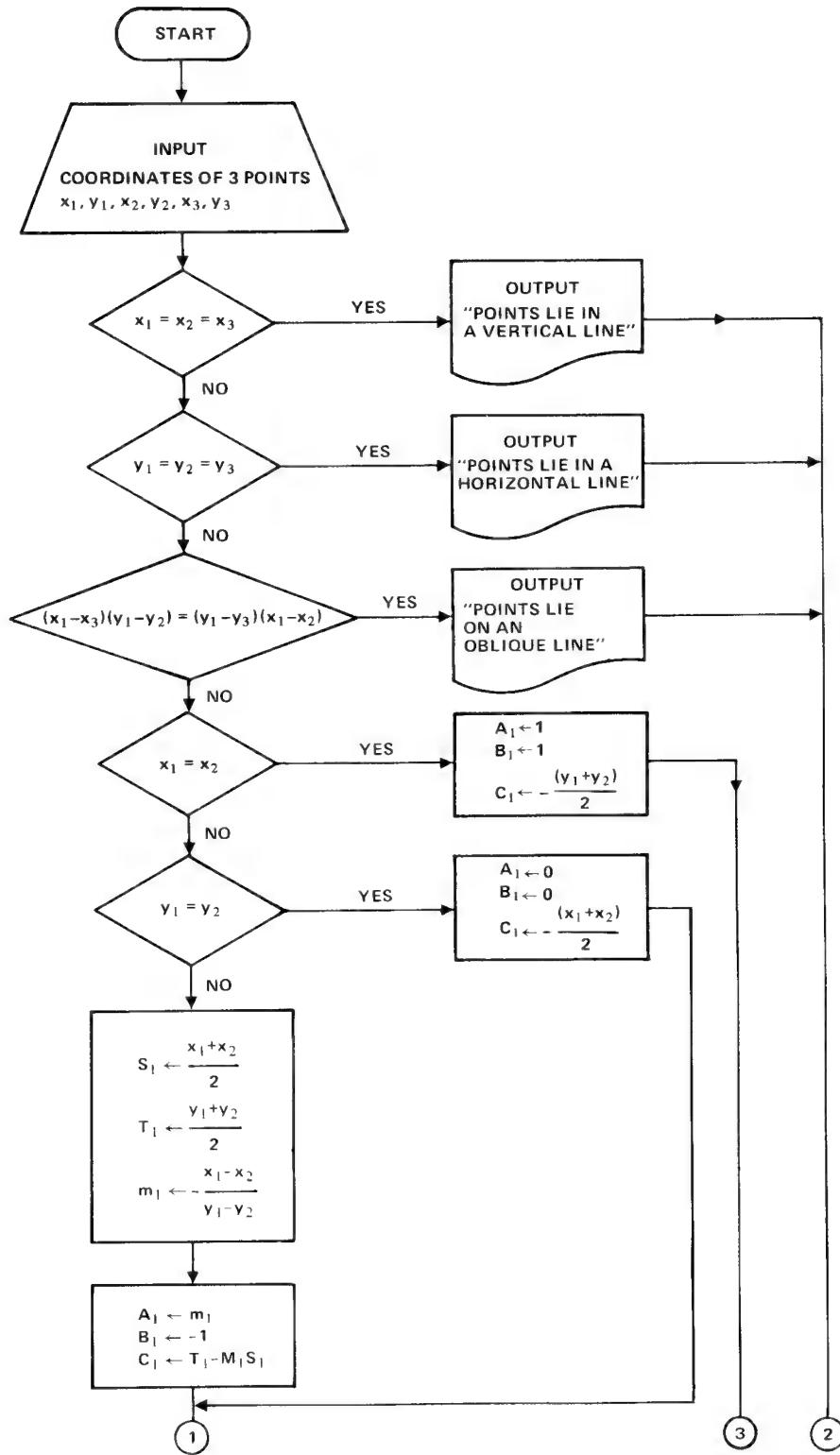
## Exercise 7 — Finding the Center and Radius of a Circle Determined by Three Points

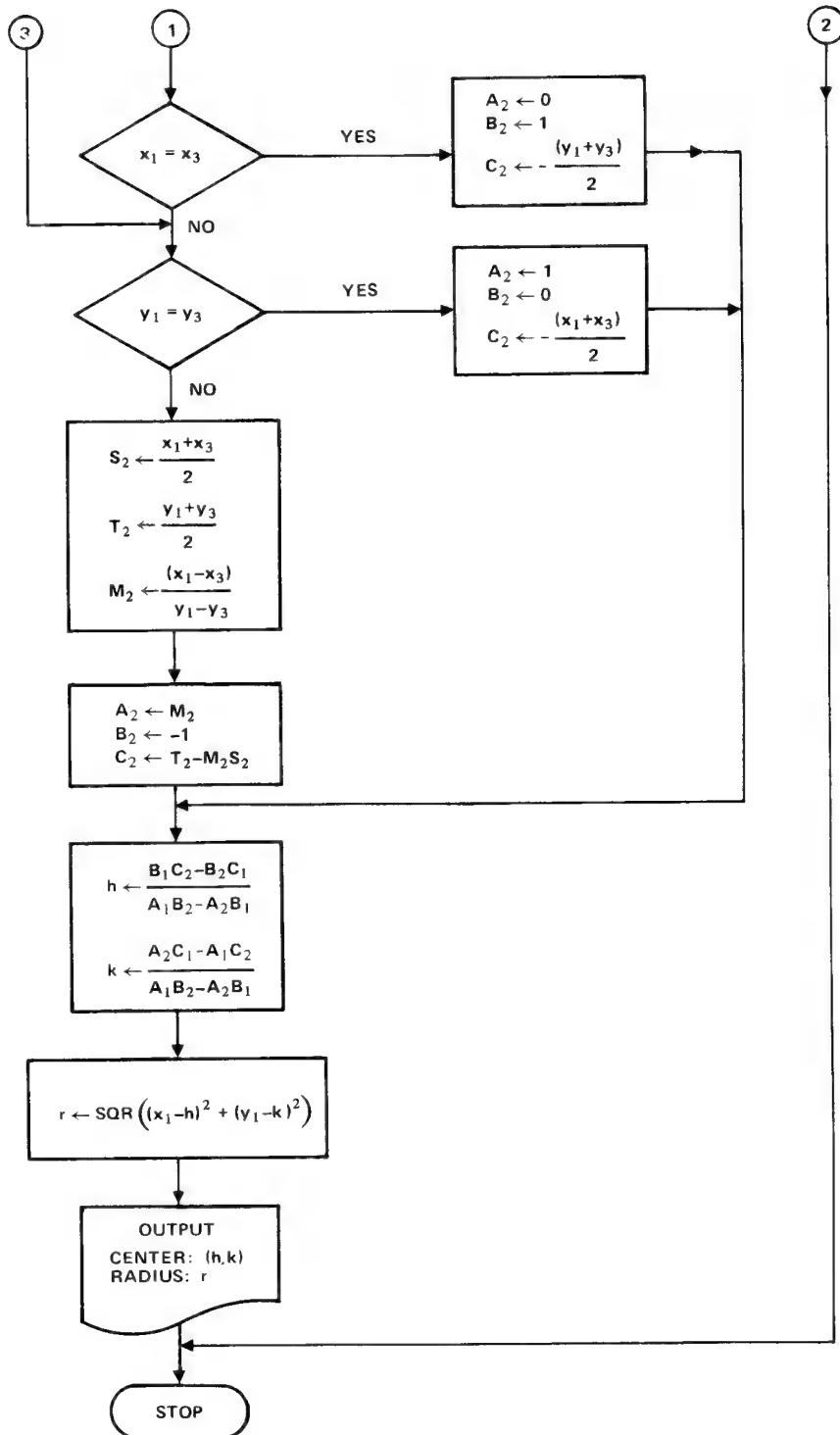
This exercise requires combining several of the previous exercises. It is expected that the first programs students write for this exercise might be rather lengthy. The flowchart and program given here are moderately detailed and lengthy, respectively. Not counting the REM lines, the example program has 53 lines.

After your students have their programs running, challenge them to reduce the code to as few lines as possible. The alternate program included here has 26 lines, but it has been written in as few as 10 lines by some students.

## Micro Flowchart

## Exercise 7.





*Example Program**Exercise 7.*

```

10 REM--A PROGRAM TO FIND THE RADIUS AND CENTER OF A CIRCLE
20 REM--DETERMINED BY THREE NON-COLLINEAR POINTS IN THE CARTESIAN
30 REM--PLANE. ON THE DATA LINE ENTER THE COORDINATES OF THE TRIPLETS
40 REM--OF POINTS TO BE CONSIDERED.
50 DATA -2,3,7,6,5,-4,6,1,-2,1,4,3,7,2,-3,-5,7,-1,3,1,3,-1,3,6
60 READ X1,Y1,X2,Y2,X3,Y3
70 IF X1=X2 AND X2=X3 THEN 300
80 IF Y1=Y2 AND Y2=Y3 THEN 330
90 IF (X1-X3)*(Y1-Y2)=(Y1-Y3)*(X1-X2) THEN 360
100 IF X1=X2 THEN 390
110 IF Y1=Y2 THEN 430
120 S1=(X1+X2)/2
130 T1=(Y1+Y2)/2
140 M1=-(X1-X2)/(Y1-Y2)
150 A1=M1
160 B1=-1
170 C1=T1-M1*S1
180 IF X1=X3 THEN 470
190 IF Y1=Y3 THEN 510
200 S2=(X1+X3)/2
210 T2=(Y1+Y3)/2
220 M2=-(X1-X3)/(Y1-Y3)
230 A2=M2
240 B2=-1
250 C2=T2-M2*S2
260 H=(B1*C2-B2*C1)/(A1*B2-A2*B1)
270 K=(A2*C1-A1*C2)/(A1*B2-A2*B1)
280 R=SQR((X1-H)^2+(Y1-K)^2)
290 GOTO 540
300 PRINT "POINTS LIE IN A VERTICAL LINE."
310 PRINT
320 GOTO 60
330 PRINT "POINTS LIE IN A HORIZONTAL LINE."
340 PRINT
350 GOTO 60
360 PRINT "POINTS LIE ON AN OBLIQUE LINE."
370 PRINT
380 GOTO 60
390 A1=0
400 B1=1
410 C1=-(Y1+Y2)/2
420 GOTO 190
430 A1=1
440 B1=0
450 C1=-(X1+X2)/2
460 GOTO 180
470 A2=0
480 B2=1
490 C2=-(Y1+Y3)/2

```

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```
500  GOTO 260
510  A2=1
520  B2=0
530  C2=-(X1+X3)/2
535  GOTO 260
540  PRINT "CENTER: (";H;",";K;"); RADIUS:";R
550  PRINT
560  GOTO 60
570  END
```

RUN

CENTER: ( 3.5 , 1.5 ); RADIUS: 5.70088

CENTER: ( 2 , -1 ); RADIUS: 4.47214

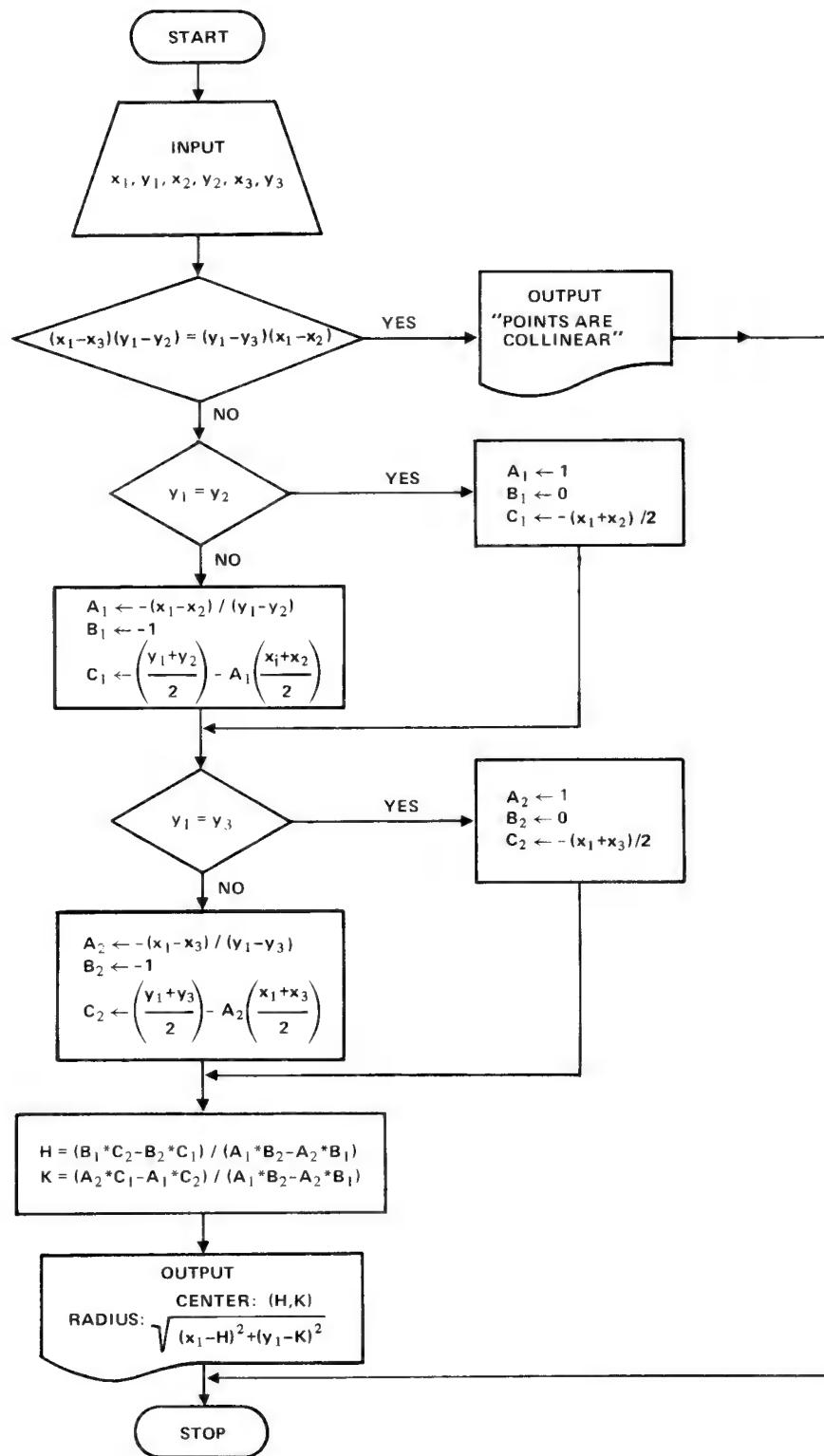
CENTER: ( .6 , .5 ); RADIUS: 6.57343

POINTS LIE IN A VERTICAL LINE.

OUT OF DATA IN LINE 60

## Alternate Flowchart

## Exercise 7.



*Alternate Program*

## Exercise 7.

```

10 REM--A PROGRAM TO FIND THE RADIUS AND CENTER OF A CIRCLE
20 REM--DETERMINED BY THREE NON-COLLINEAR POINTS IN THE CARTESIAN
30 REM--PLANE. ON THE DATA LINE ENTER THE COORDINATES OF THE TRIPLETS
40 REM--OF POINTS TO BE CONSIDERED.
50 DATA -2,3,7,6,5,-4,6,1,-2,1,4,3,7,2,-3,-5,7,-1,3,1,3,-1,3,6
60 READ X1,Y1,X2,Y2,X3,Y3
70 IF (X1-X3)*(Y1-Y2)=(Y1-Y3)*(X1-X2) THEN 200
80 IF Y1=Y2 THEN 220
90 A1=-(X1-X2)/(Y1-Y2)
100 B1=-1
110 C1=(Y1+Y2)/2-A1*(X1+X2)/2
120 IF Y1=Y3 THEN 260
130 A2=-(X1-X3)/(Y1-Y3)
140 B2=-1
150 C2=(Y1+Y3)/2-A2*(X1+X3)/2
160 H=(B1*C2-B2*C1)/(A1*B2-A2*B1)
170 K=(A2*C1-A1*C2)/(A1*B2-A2*B1)
180 PRINT "CENTER: (";H;",";K;")"; RADIUS: ";SQR((X1-H)^2+(Y1-K)^2)
190 GOTO 60
200 PRINT "POINTS ARE COLLINEAR"
210 GOTO 60
220 A1=1
230 B1=0
240 C1=-(X1+X2)/2
250 GOTO 120
260 A2=1
270 B2=0
280 C2=-(X1+X3)/2
290 GOTO 160
300 END

```

RUN

```

CENTER: ( 3.5      , 1.5      ) ; RADIUS:  5.70088
CENTER: ( 2        , -1       ) ; RADIUS:  4.47214
CENTER: ( .6       , .5       ) ; RADIUS:  6.57343
POINTS ARE COLLINEAR

```

OUT OF DATA IN LINE 60

**LINEAR SYSTEMS****Exercise 8 – The Interative Method**

This exercise is an introduction to an iterative process that can be applied to non-linear systems as well as to linear systems. Students should be encouraged to investigate the method further by studying the suggested references.

The flowchart in the Student Book is sufficient for writing the following program. We have used the DEF FN (defining a function) statement in this program. If your students have not used this statement, this will give them a good introduction to its use.

*Example Program**Exercise 8.*

```

10  REM--A PROGRAM TO FIND THE SOLUTION OF A SECOND ORDER LINEAR
11  REM--SYSTEM BY AN ITERATIVE PROCESS.  ON THE DATA LINE ENTER
12  REM--THE DESIRED SOLUTION TOLERANCE T, AND THE COEFFICIENTS OF
13  REM--THE GENERAL FORM OF THE SYSTEM.
14  DIM X[200],Y[200]
20  DATA .0001,39,1,-6,25,22,-13,.0001,2.135,86.93,.7394,.8113,4.793
21  DATA -6.843
30  READ T,A1,B1,C1,A2,B2,C2
35  M1=-A1/B1
40  M2=-A2/B2
50  DEF FNA(X)=(C1-A1*X)/B1
60  DEF FNB(Y)=(-C1-B1*Y)/A1
70  DEF FNC(X)=(-C2-A2*X)/B2
80  DEF FND(Y)=(-C2-B2*Y)/A2
90  I=1
100  X[I]=1
110  IF ABS(M1)>ABS(M2) THEN 180
120  Y[I]=FNA(X[I])
130  I=I+1
140  X[I]=FND(Y[I-1])
150  Y[I]=FNA(X[I])
160  IF ABS(X[I]-X[I-1])<T OR ABS(Y[I]-Y[I-1])<T THEN 230
170  GOTO 130
180  Y[I]=FNC(X[I])
190  I=I+1
200  X[I]=FNB(Y[I-1])
210  Y[I]=FNC(X[I])
220  IF ABS(X[I]-X[I-1])<T OR ABS(Y[I]-Y[I-1])<T THEN 230
225  GOTO 190
230  PRINT "(";X[I];",";Y[I];")"
235  GOTO 30
240  END

```

RUN

```

( .142858      , .428571      )
( 9.8068       ,-.232349      )

```

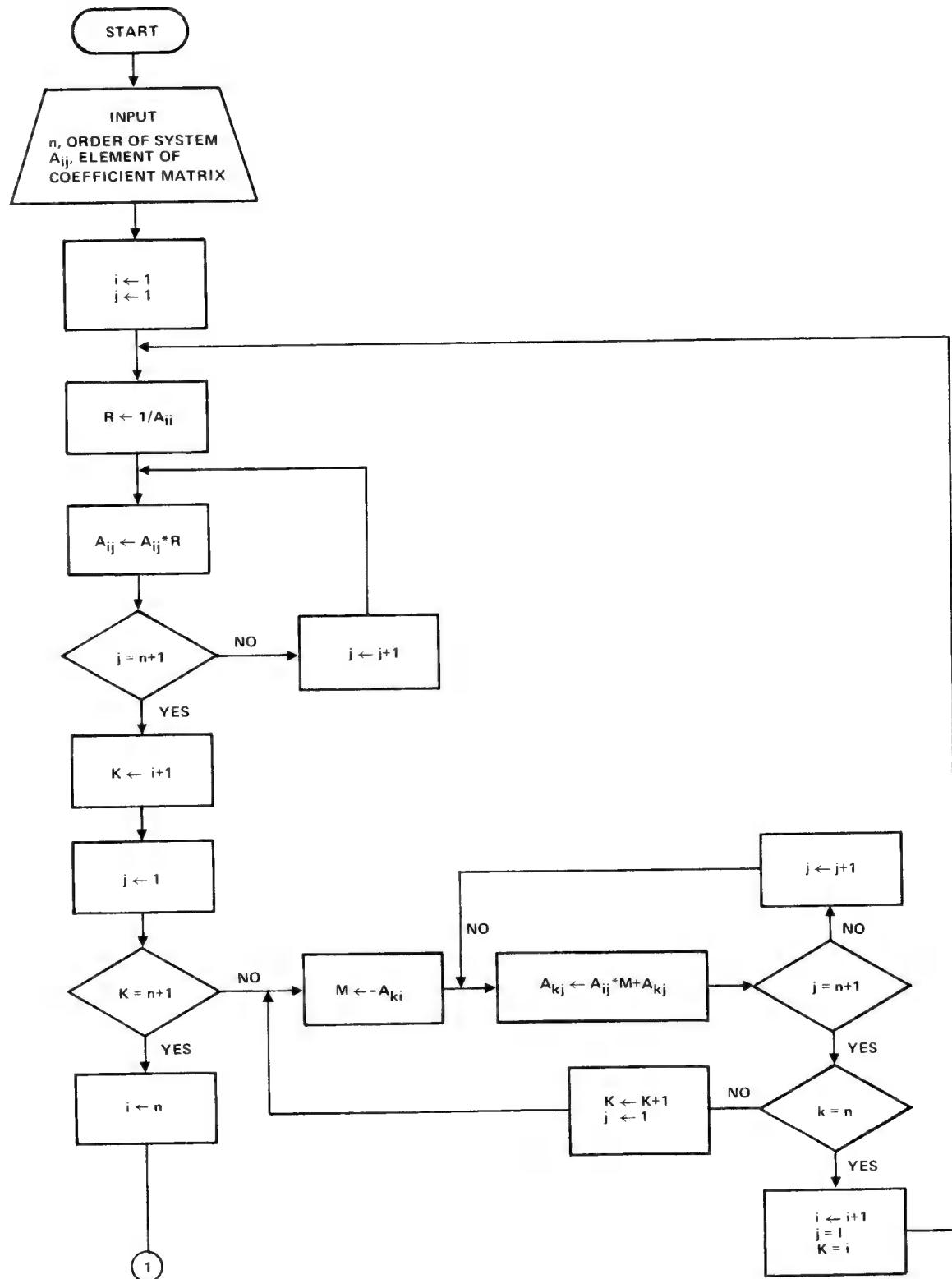
OUT OF DATA IN LINE 30

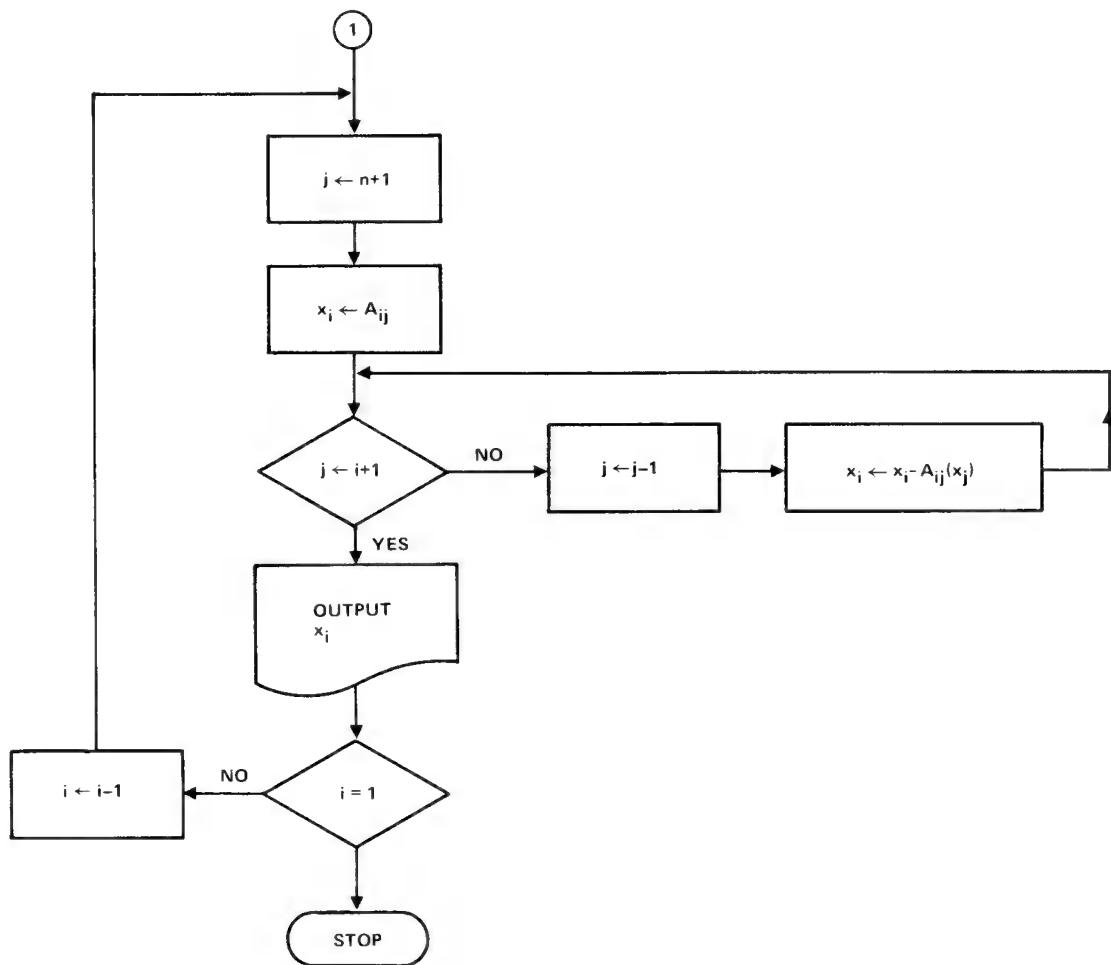
**Exercise 9 — Applying the Gaussian Algorithm**

The discussion of the Gaussian Algorithm in the Student Book is detailed and thorough as far as it goes. No attempt is made to deal with round-off error and other problems which might be encountered. Encourage your students to study the suggested reference for a more extensive treatment of this topic.

### *Micro Flowchart*

*Exercise 9(a).*





*Example Program*

*Exercise 9(a).*

```

10 REM--A PROGRAM TO SOLVE AN NTH ORDER LINEAR SYSTEM OF EQUATIONS
11 REM--USING THE GAUSSIAN ALGORITHM TO FORM A TRIANGULAR COEFFECIENT
12 REM--MATRIX AND BACK SOLVING PROCESS TO FIND THE SOLUTION.
13 REM--ON THE DATA LINE ENTER THE ORDER NUMBER OF THE SYSTEM FOLLOWED
14 REM--BY THE ELEMENTS OF THE COEFFICIENT MATRIX.
50 DATA 2,39,1,6,25,22,13,2,2.135,86.93,-.7394,.8113,4.793
51 DATA 6.843,3,2,-1,3,1,-1,3,-2,0,1,1,1,-2,4,3,7,-2,-1,2,4,-2,1
52 DATA -3,5,-2,2,-5,2,-1,4,-1,3,1,3
80 READ N
90 FOR I=1 TO N
100 FOR J=1 TO N+1
110 READ A[I,J]
120 NEXT J
130 NEXT I
140 I=J=1
150 R=1/A[I,I]
  
```

```

160  A[I,J]=A[I,J]*R
170  IF J=N+1 THEN 200
180  J=J+1
190  GOTO 160
200  K=I+1
210  J=1
220  IF K=N+1 THEN 360
230  M=-A[K,I]
240  A[K,J]=A[I,J]*M+A[K,J]
250  IF J=N+1 THEN 280
260  J=J+1
270  GOTO 240
280  IF K=N THEN 320
290  K=K+1
300  J=1
310  GOTO 230
320  I=I+1
330  J=I
340  K=I
350  GOTO 150
360  I=N
370  J=N+1
380  X[I]=A[I,J]
390  IF J=I+1 THEN 420
400  J=J-1
410  X[I]=X[I]-A[I,J]*(X[J])
415  GOTO 390
420  PRINT "X(";I;") =";X[I]
430  IF I=1 THEN 455
440  I=I-1
450  GOTO 370
455  PRINT
456  GOTO 80
460  END

```

RUN

```

X( 2      ) = .428571
X( 1      ) = .142857

```

```

X( 2      ) =-.252262
X( 1      ) = 9.92492

```

```

X( 3      ) = 14.
X( 2      ) = 3
X( 1      ) =-19.

```

```

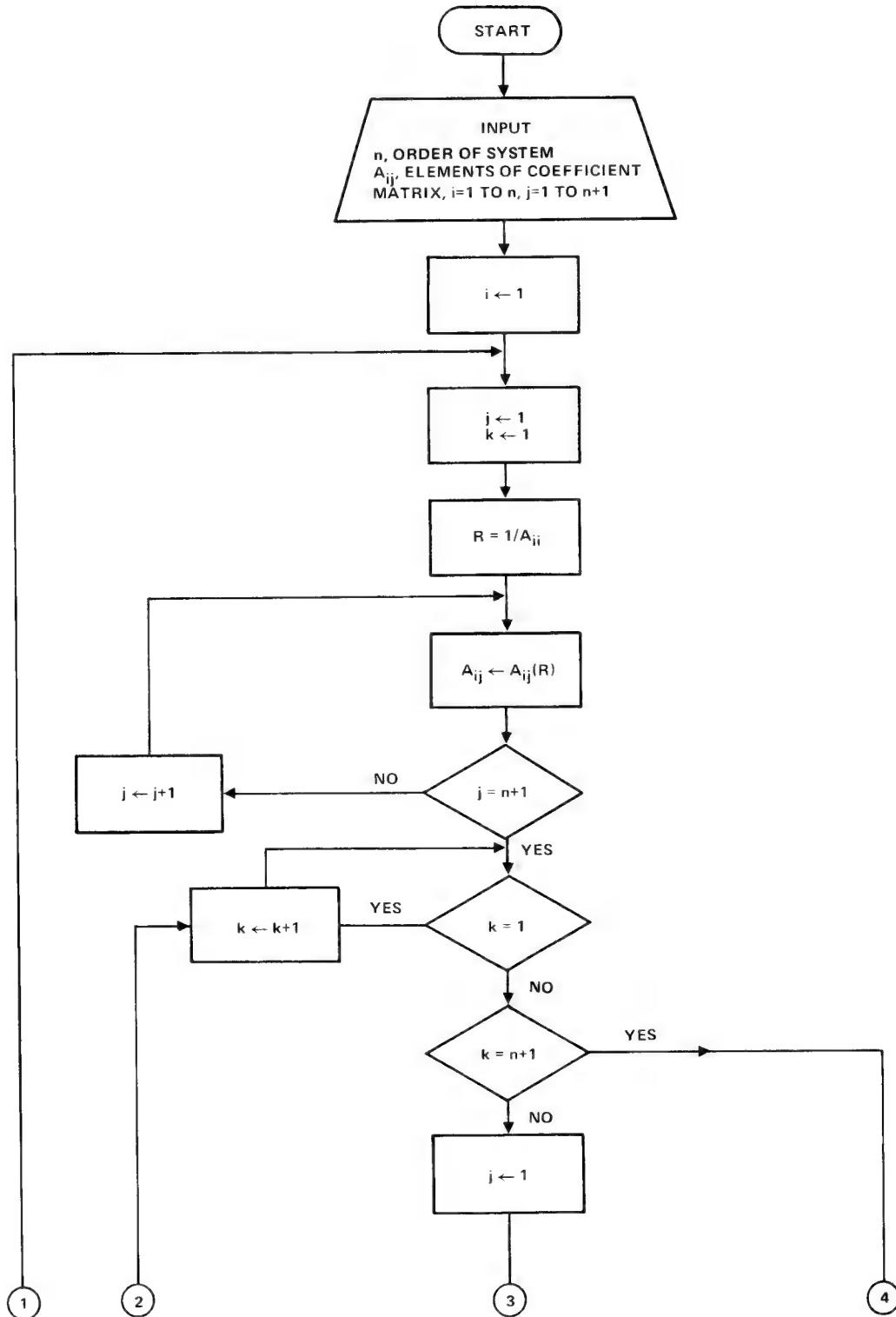
X( 4      ) =-.208696
X( 3      ) =-.426087
X( 2      ) =-.313044
X( 1      ) = 1.04348

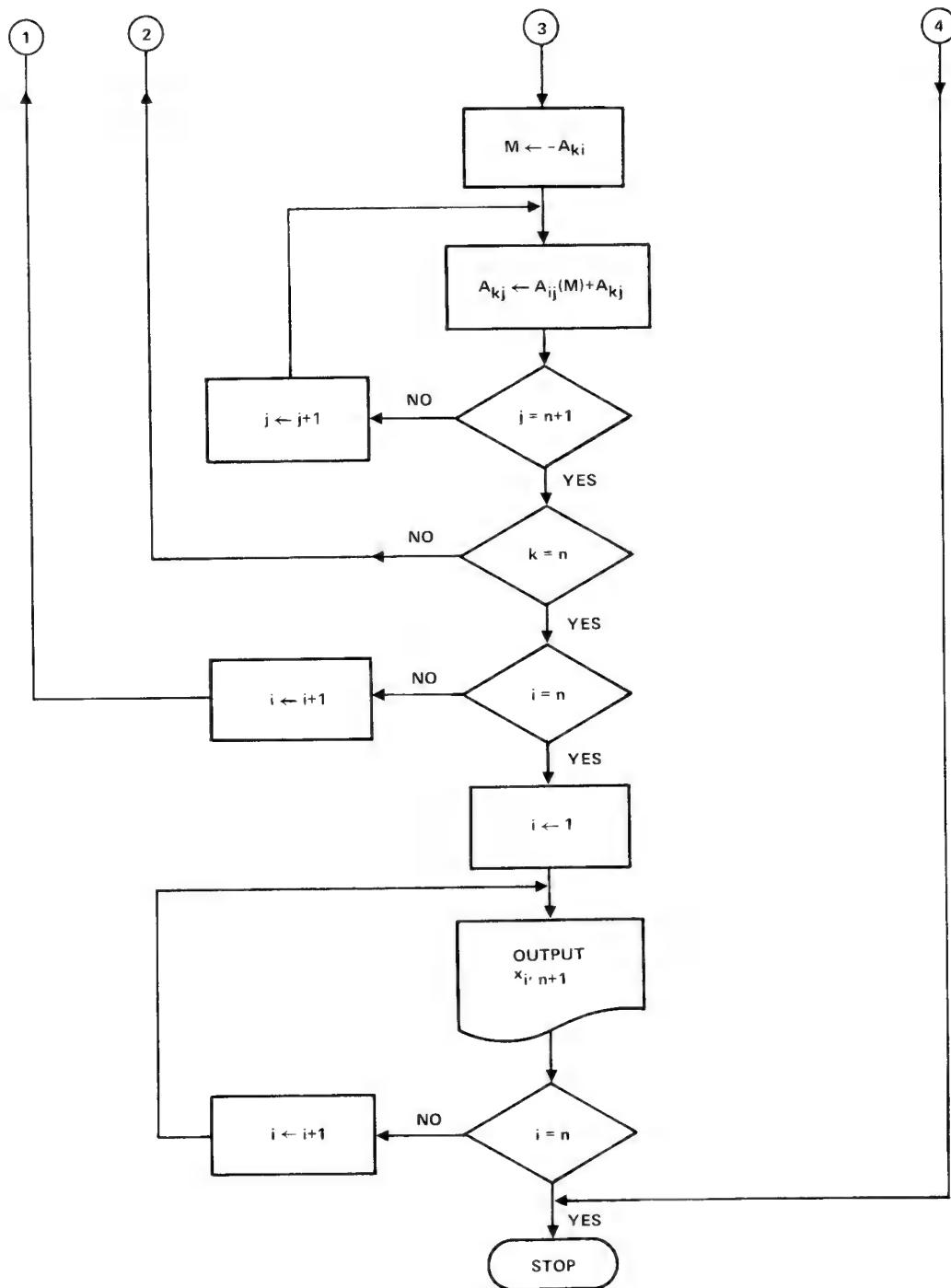
```

OUT OF DATA IN LINE 80

## Macro Flowchart

## Exercise 9(b).





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#### Example Program

##### Exercise 9(b).

```
10  REM--A PROGRAM THAT SOLVES A SYSTEM OF EQUATIONS BY THE GAUSS-
20  REM--JORDAN ELIMINATION METHOD.  ON THE DATA LINE, ENTER,N,THE
30  REM-- ORDER OF THE SYSTEM, AND THE COEFFECIENT MATRIX
40  REM--ROW BY ROW.
50  DATA 2,39,1,6,25,22,13,2,2.135,86.93,-.7394,.8113,4.793,6.843
60  DATA 3,2,-1,3,1,-1,3,-2,0,1,1,1,-2,4,3,7,-2,-1,2,4,-2,1,-3,5,-2,2,-8
70  DATA -1,4,-1,3,1,3
80  READ N
90  FOR I=1 TO N
100  FOR J=1 TO N+1
110  READ A[I,J]
120  NEXT J
130  NEXT I
140  I=1
150  J=1
160  K=1
170  R=1/A[I,I]
180  A[I,J]=A[I,J]*R
190  IF J=N+1 THEN 220
200  J=J+1
210  GOTO 180
220  IF K=I THEN 310
230  IF K=N+1 THEN 360
240  J=1
250  M=-A[K,I]
260  A[K,J]=A[I,J]*M+A[K,J]
270  IF J=N+1 THEN 300
280  J=J+1
290  GOTO 260
300  IF K=N THEN 330
310  K=K+1
320  GOTO 220
330  IF I=N THEN 360
340  I=I+1
350  GOTO 150
360  FOR I=1 TO N
370  PRINT "X(";I;")=";A[I,N+1]
380  NEXT I
385  GOTO 80
390  END
```

RUN

```
X( 1 ) = .142857
X( 2 ) = .428571
X( 1 ) = 9.92492
X( 2 ) = -.252262
X( 1 ) = -19.
X( 2 ) = 3
X( 3 ) = 14.
X( 1 ) = 1.04348
X( 2 ) = -.313044
X( 3 ) = -.426087
X( 4 ) = -.208696
```

OUT OF DATA IN LINE 80

**Exercise 10 — Solving Linear Systems by Use of Matrices**

The following program can be written directly from the flowchart in the Lab Book.

*Example Program**Exercise 10.*

```

10 REM--A PROGRAM TO SOLVE LINEAR SYSTEMS USING MATRICES.
20 REM--ON THE DATA LINE ENTER THE ORDER OF THE SYSTEM,
30 REM--FOLLOWED BY THE COEFFICIENTS OF THE SYSTEM.
40 DATA 2,39,1,25,22,6,13
50 DATA 2,2.1356,86.93,.8113,4.793,-.7394,6.843
60 DATA 3,2,-1,3,-1,3,-2,1,1,1,1,0,-2,4,3,7,-3,-1,4,-2,1,-3
70 DATA -2,2,-5,2,4,-1,3,1,2,5,-1,3
80 READ N
90 MAT READ A[N,N],K[N,1]
100 MAT S=ZER[N,1]
110 MAT I=ZER[N,N]
120 MAT I=INV(A)
130 MAT S=I*K
140 PRINT "THE SOLUTION TO THE SYSTEM IS:"
150 MAT PRINT S
160 GOTO 80
170 END

```

RUN

THE SOLUTION TO THE SYSTEM IS:

.142857

.428571

THE SOLUTION TO THE SYSTEM IS:

9.9254

-.252342

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THE SOLUTION TO THE SYSTEM IS:

- 19 .

3

14

THE SOLUTION TO THE SYSTEM IS:

1.03209

- .370767

- .43672

- .188948

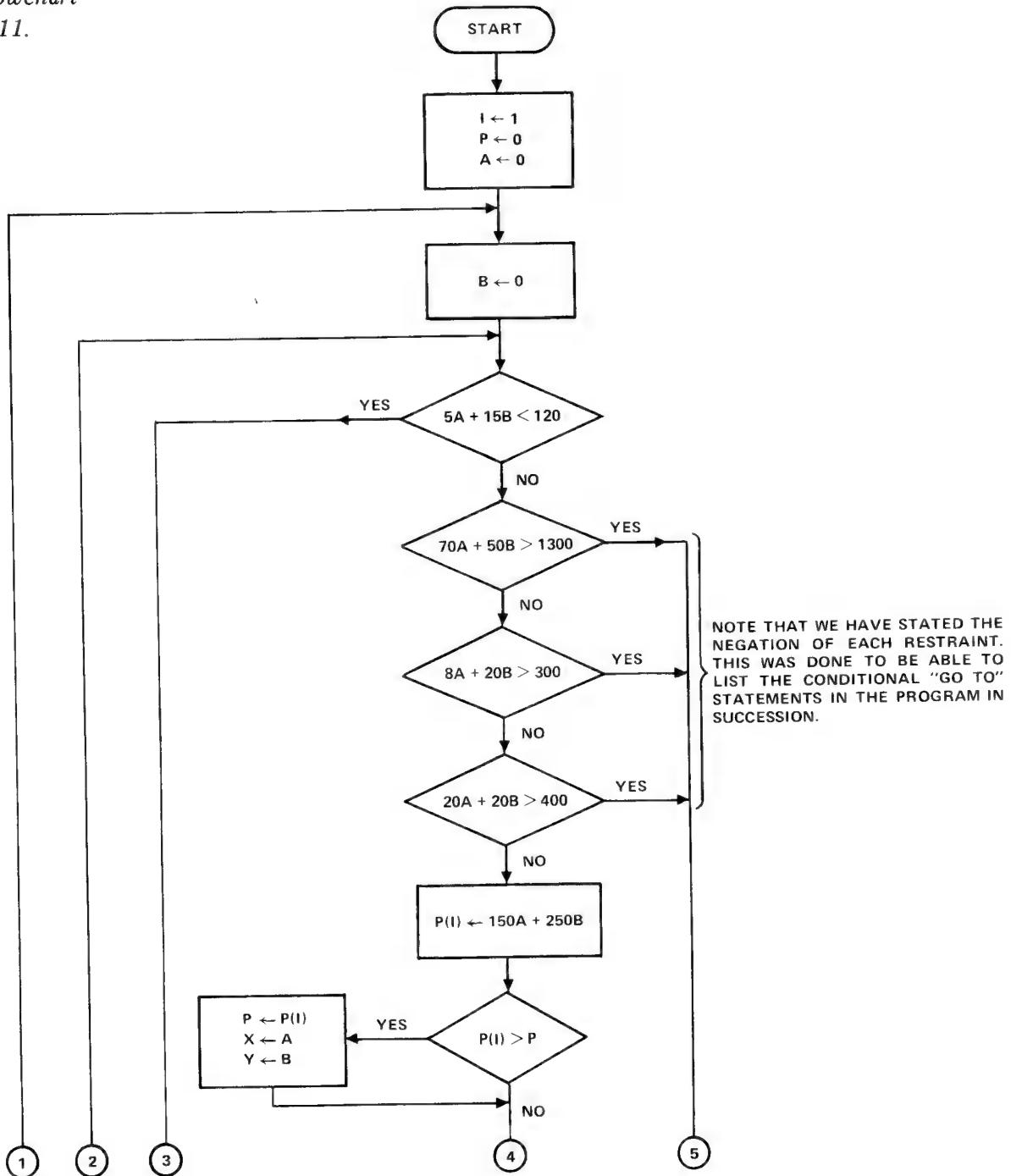
OUT OF DATA IN LINE 80

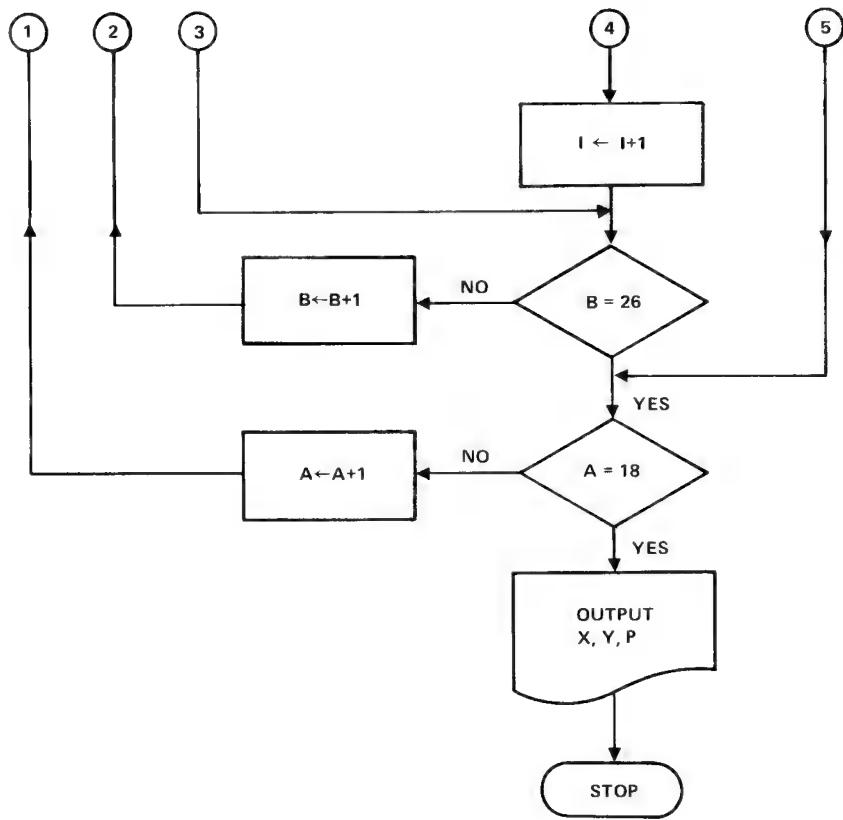
## LINEAR PROGRAMMING

## Exercise 11 – A Manufacturing Problem

## Micro Flowchart

## Exercise 11.





*Example Program**Exercise 11.*

```

10 REM--A PROGRAM TO SOLVE A LINEAR , TWO VARIABLE , PROGRAMMING
11 REM--PROBLEM. ON THE DATA LINE ENTER THE UPPER BOUNDS, A AND B,
12 REM--OF THE TWO VARIABLES. ON LINE 60 ENTER THE
13 REM--CONSTRAINTS ON THE VARIABLES.
14 DIM P[100]
40 P=A=0
50 I=1
55 B=0
60 IF 5*A+15*B<120 THEN 130
70 IF 70*A+50*B>1300 THEN 200
80 IF 8*A+20*B>300 THEN 200
90 IF 20*A+20*B>400 THEN 200
100 P[I]=150*A+250*B
110 IF P[I]>P THEN 160
120 I=I+1
130 IF B=26 THEN 200
140 B=B+1
150 GOTO 60
160 P=P[I]
170 X=A
180 Y=B
190 GOTO 120
200 IF A=18 THEN 230
210 A=A+1
220 GOTO 55
230 PRINT "THE MAXIMUM PROFIT ";P;"IS OBTAINED FOR MANUFACTURING"
240 PRINT X;"LOTS OF TYPE A TOYS AND";Y;"LOTS OF TYPE B TOYS."
250 END

```

RUN

THE MAXIMUM PROFIT 4100 IS OBTAINED FOR MANUFACTURING  
 9 LOTS OF TYPE A TOYS AND 11 LOTS OF TYPE B TOYS.

DONE

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**NOTES**

## NOTES

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